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An experimental investigation into forced, natural and combined forced and natural convective heat transfer from stationary isothermal circular disks

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Abstract—Experimental heat transfer data are presented and dimensionless correlations developed for forced, natural and combined assisting forced and natural convection for heated stationary isothermal circular disks over wide ranges of the Reynolds, Rayleigh and modified Reynolds numbers, respectively. Experiments with air were performed for a variety of disks ranging in diameter and thickness-to-diameter aspect ratio. The correlation for combined forced and natural convection was developed utilizing the concept of a modified Reynolds number which accounts for a buoyancy-induced velocity. Utilizing this concept, the experimental data and respective empirical correlations for all three convection modes can be collapsed and plotted on the same continuous curve.

INTRODUCTION

Considerable empirical data exist in the literature for forced convection heat transfer involving external flow over a variety of geometries, and for various ranges of Reynolds number. Many current heat transfer textbooks [1–5] present empirical correlations for forced external flow over a flat plate, a sphere, cylinders, both aligned and staggered cylinder bundles in cross flow, and for tubes of cylindrical, hexagonal, square and other assorted cross-sections, as well as falling drops and packed sphere beds. Furthermore, empirical correlations exist for natural convection‡ heat transfer from geometries such as vertical, horizontal and inclined flat plates, horizontal and vertical cylinders, spheres, bispheres, oblate and prolate spheroids, horizontal upward and downward facing surfaces, cubes of various orientations, vertical and inclined channels, rotating geometries, as well as geometries within enclosures, and over a wide range of the Rayleigh number. In the area of combined forced and natural convection, it appears that most of the attention has been focused on vertical and horizontal flat plates and cylinders. A geometry that seems to be missing from all of these lists is that of a thin circular disk. The disk-type geometry is relevant in the cooling of electronic components, such as disk-shaped resistors and power transistors, and the use of disk-type thermistors for temperature and air flow measurements.

Some experimental and theoretical studies have been carried out for natural convection from horizontal and inclined disk surfaces, however only limited experimental data exist in the available literature for forced [7] or natural [8] convective heat transfer for circular disks, and there appears to be no existing data for this geometry under conditions of combined forced and natural convection.

There have been some experimental research [9–11] and theoretical studies [12, 13] devoted to natural convection heat transfer from stationary and rotating horizontal circular disk§ surfaces. Hassani and Hollands [8] performed experiments measuring the natural convection heat transfer from a circular disk in both vertical and horizontal configurations,¶ and proposed a characteristic length‖ such that the experimental data obtained could be collapsed with certain other shapes for a limited range of the Rayleigh number, the goal being a type of 'universal correlation'. With the exception of this 'universal correlation', to the best knowledge of the authors, no other empirical correlation currently exists in the available

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‡ The implications of buoyancy-induced flows as they pertain to applications in technology are given by Gebhart [6].

§ The term circular disk in these references is for a single circular disk *surface* only (top or bottom) and not of the entire three-dimensional circular disk.

¶ The term vertical will refer to the configuration such that the direction of the buoyancy force is *perpendicular* to the axis of the disk, and thus parallel to the two flat sides. The term horizontal refers to the configuration where the direction of the buoyancy force is *parallel* to the axis of the disk, and thus perpendicular to the two flat sides.

‖ Other investigators have strived, with limited success, to obtain a characteristic length such that experimental data for a variety of shapes could be collapsed to a single curve. The interested reader should refer to the works of Sparrow and Ansari [14], Sparrow and Stretton [15] and Lienhard [16] as well as Hassani and Hollands [8].

NOMENCLATURE

A	total thermistor heat transfer area [m^2]
C	empirically determined coefficient
d	diameter of circular disk heat transfer model [m]
Gr_d	Grashof number, $\rho^2 g \beta'(T-T_f)d^3/\mu^2$
h	average convective heat transfer coefficient [W m^{-2} K^{-1}]
n	empirically determined exponent
Nu_d	average Nusselt number, hd/k
Pr	Prandtl number, $\mu c_p/k$
$\dot{Q}$	convective heat transfer rate from thermistor heat transfer model [W]
R	thermistor heat transfer model resistance [Ω]
Ra_d	Rayleigh number, $Pr\,Gr_d$
Re_d	Reynolds number, $\rho v_f d/\mu$
Re_d^*	modified Reynolds number, $Re_d+\lambda(Gr_d/2)^{1/2}$
Ri_d	Richardson number, Gr_d/Re_d^2
R_s	standard resistance [Ω]
t	thickness of circular disk heat transfer model [m]
T	thermistor heat transfer model temperature [K]
$u_{n,max}$	maximum local velocity in natural convection boundary layer [m s^{-1}]
$\bar{u}_{n,max}$	average maximum velocity in natural convection boundary layer [m s^{-1}]
V	voltage drop across thermistor [V]
v_f	free-stream fluid velocity [m s^{-1}]
V_s	voltage drop across standard resistor [V].

Greek symbols

α	thermistor calibration constant [Ω]
β	thermistor calibration constant [K]
β'	coefficient of thermal expansion [K^{-1}]
λ	natural convection weighting factor in the modified Reynolds number.

Subscripts

n	natural convection
f	forced convection
c	combined forced and natural convection.

literature for natural convection from stationary vertical thin circular disks in the familiar standard form

$$Nu_d = C_n(Pr\,Gr_d)^{n_n}. \quad (1)$$

This is one of the objectives of the current research.

The mode of convection which is neither dominated by pure forced nor pure natural convection, but is rather a combination of the two, is appropriately referred to as combined, or mixed, forced and natural convection. In such a situation, the relative direction of the buoyancy force and the externally forced flow is important. In the case where the external forced flow is in the same direction as that of the buoyancy force, the mode of thermal energy transport is termed assisting (or aiding) combined convection. Similarly, in the case where the forced flow is in a direction directly opposite that of the buoyancy force, the mode of energy transport is termed opposing combined convection. A third, less common situation occurs when the directions of the forced fluid flow and the buoyancy force are perpendicular to one another, in which case the mode of thermal energy transport is termed traverse combined convection. It is generally accepted that the primary dimensionless parameter influencing combined forced and natural convection phenomena is the Richardson number, Ri_x, which comes about directly from the dimensionless form of the Navier–Stokes equation [17] and is defined as

$$Ri_x \equiv \frac{Gr_x}{Re_x^2}. \quad (2)$$

Most current heat transfer texts [1–5, 18] note the Richardson number as the primary parameter. However, although this parameter is widely accepted, there is still some question in the literature as to whether it represents the best parameter. For example, Churchill [19] proposed the use of $Ri_x/Pr^{1/3}$ so as to not confine the work to a single fluid. Acrivos [20] noted that the Richardson number is the primary parameter for fluids whose Prandtl number, Pr, is less than unity, and $Ri_x/Pr^{1/3}$ is the primary parameter for fluids whose Prandtl number is much greater than unity. Wilks [21] suggested that the parameter $Ri_x/Pr^{1/3}$ may be used with some degree of confidence for fluids with Prandtl numbers as low as 0.4. This same parameter appears in research by Tsuruno and Iguchi [22].

There appears to have been relatively little research done on combined forced and natural convection as compared with either pure forced or pure natural convection, a good summary of the existing literature being given by Churchill [19] and more recently by Gebhart *et al.* [4]. A common coupling rule [4, 5] for combined forced and natural convection is an addition of correlations for forced and natural convection, each correlation raised to the same power, i.e.

$$Nu^n = Nu_F^n \pm Nu_N^n \quad (3)$$

where the subscripts F and N refer to pure forced and pure natural convection respectively, and the exponent, n, is determined from experimental data and varies from one geometry to the next. The addition of the forced and natural terms in equation (3) is used in the case of assisting flow, while a difference of the terms is used in opposing flow. As will be discussed later, other 'rules' for combined forced and natural convection include replacing the buoyancy force by a pseudo-velocity [23, 24] and, more notably, by utilizing a modified Reynolds number, Re_x^*, which incorporates a characteristic buoyancy-induced velocity along with the free-stream velocity. The resulting modified Reynolds number is of the form

$$Re_x^* = Re_x + \lambda\sqrt{(Gr_x/2)}. \quad (4)$$

However, there is a lack of consensus in the available literature on the value of the weighting factor, λ. Although the concept of a modified Reynolds number is less common than the combining rule of equation (3), it will be shown later that the modified Reynolds number is very effective in the current research for developing an empirical correlation for combined forced and natural convective heat transfer.

Although there have been a limited number of experimental and theoretical studies of a few simple external geometries for combined forced and natural convection, there have not, to the best knowledge of the authors, been any published studies for circular disks. Most of the experimental studies for combined forced and natural convection for external flows have been performed on long cylinders and flat plates. However, even for such a familiar geometry as a vertical flat plate, only limited experimental data exist for assisting flow, and even fewer for opposing flow. The experimental data available, such as that presented by Kliegel [25], Gryzagoridis [26], Oosthuizen and Hart [27], Lloyd and Sparrow [28] and Ramachandran *et al.* [29], are for local heat transfer, the data being obtained by measuring, usually with an interferometer, local temperature gradients. In addition, Oosthuizen and Bassey [30] presented experimental data for the average heat transfer for a vertical flat plate under conditions of both assisting and opposing flow.

EXPERIMENTAL APPARATUS AND MEASUREMENT TECHNIQUES

The circular disks that were used as heat transfer models for the experimental data presented in this paper were commercially available disk-type thermistors.† Six different circular disk models were tested, ranging in diameter, d, from 5.21 to 19.99 mm and in thickness-to-diameter aspect ratio, t/d, between 0.058 and 0.2. Thermistors were chosen as the heat transfer models because they provided a unique combination for indirectly measuring the surface temperature and the convective heat transfer rate [7, 31]. The thermistor was self-heated by means of Joule heating. Conduction losses through the thermistor lead wires (0.127 mm diameter) were minimized (less than 3, 6 and 8%‡ for forced, combined forced and natural, and natural convection, respectively) by using constantan wire, which has a low enough thermal conductivity to minimize the 'fin effect' and an electrical resistivity low enough to minimize Joule heating. Also, a one-dimensional analysis was done on the lead wires modeling the 'fin effect' and taking into account the possibility of Joule heating; the result of this analysis indicated that the existence of any Joule heating acted to decrease the conduction losses through the lead wires.

Using an electrical circuit suggested by Wedekind [7], as shown in Fig. 1, the thermistor current and resistance can be accurately and simultaneously measured during self-heating. This makes it possible to measure indirectly not only the convective heat transfer rate, but also the average temperature of the thermistor, the latter by having pre-calibrated the resistance/temperature characteristics of each thermistor heat transfer model. Thermistors have a high resistance coefficient, therefore the heat transfer surface temperature, T, could be indirectly measured quite accurately§ without the many difficulties encountered in attempting to measure the surface temperature (especially on small heat transfer models) by conventional means.¶

† Thermistors are semiconductors of ceramic material made by sintering mixtures of metallic oxides such as manganese, nickel, cobalt, copper, iron and uranium. Disks are made by pressing thermistor material under high pressure in a round die to produce flat coin-like pieces. These pieces are then coated with silver on the two flat surfaces. Refer to *Thermistor Manual*, Fenwal Electronics, Framingham, MA 01701, U.S.A.

‡ The maximum errors occurred in the smallest of the heat transfer models ($d = 5.21$ mm) where the size of the lead wire was proportionately larger in comparison with the diameter of the disk thermistor.

§ This assumes the temperature to be uniform within and over the surface of the thermistor. Since the thermistor is thin, with a uniform voltage difference between its flat surfaces, the current flux will be uniform, resulting in a uniform Joule heating within the disk. Therefore, a simple one-dimensional thermal analysis indicates a worst-case temperature difference between the center plane and the flat surfaces of the disk to be less than 1°C for the range of experimental data presented. Radial temperature variation, due to local variation in the convective boundary flux, will also be small because the thermal conductivity of the thermistor material is sufficiently high.

¶ The experimental measurement difficulties alluded to here are those associated with attempting to measure surface temperature by mounting a temperature sensor such as a thermocouple on the surface. The presence of the thermocouple distorts the true surface temperature to varying degrees, depending upon the relative size of the heat transfer surface and the diameter of the thermocouple wire. The major mechanism for this distortion is the conduction fin effects within the thermocouple wires, which would tend to lower the temperature of a heated surface at the point of measurement.

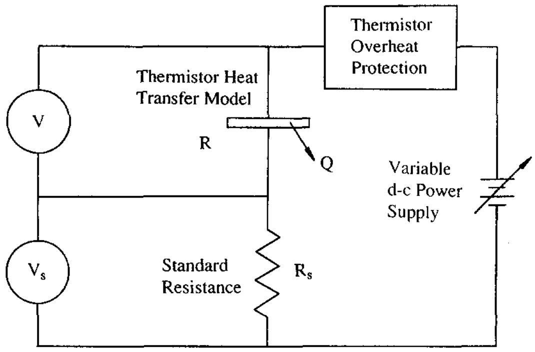

Fig. 1. Schematic of electric circuit network; indirect measurement of thermistor temperature and convective heat transfer rate.

The average convective heat transfer coefficient, h, can be expressed in terms of the heat transfer rate, $\dot{Q}$, the heat transfer area, A, and the temperature difference, $(T-T_f)$, between the surface and the fluid; thus

$$h = \frac{\dot{Q}}{A(T-T_f)} \quad (5)$$

where, as mentioned earlier, the convective heat transfer rate, $\dot{Q}$, can be indirectly measured [7] by analyzing the electrical circuit represented in Fig. 1. Thus

$$\dot{Q} = \frac{(VV_s)}{R_s}. \quad (6)$$

The disk surface temperature, T, is indirectly measured by utilizing the exponential relationship that exists between the thermistor resistance, R, and its absolute temperature, T, which is of the form

$$R = R(T) = \alpha\, e^{\beta/T} \quad (7)$$

where the parameters α and β are unique for a given thermistor, and can be determined from calibration measurements. Once these parameters are known for a given thermistor heat transfer model, the thermistor temperature, T, can be readily determined by rearranging equation (7) and making appropriate substitutions for the thermistor resistance, R; thus

$$T = \frac{\beta}{\ln[(V/V_s)(R_s/\alpha)]}. \quad (8)$$

A constant temperature may be maintained during a test by adjusting the power supply voltage such that the thermistor-standard resistor voltage ratio, V/V_s, remains constant.

† Because of the low velocities encountered in combined forced and natural convection experimentation, a pitot tube could not be used. Therefore, variable-area flowmeters (the smallest of which had a full-scale reading of 7.87 $cm^3\ s^{-1}$) were used upstream of the diffuser section to measure the volumetric flowrate and thus indirectly the average velocity. Also, a hot-wire anemometer was used to measure the velocity distribution in the test section, the non-uniformity of which was less than 10% at all times.

A schematic of the experimental apparatus, which amounts to a miniature wind tunnel made possible by the small size of the thermistor heat transfer models, is shown in Fig. 2, and is the same as that described by Wedekind [7], except that it is in the vertical position. Since the inclination angle of the apparatus is not relevant when the pure forced convection experimentation was performed, the forced convection experimentation was done with the apparatus in the horizontal position [7]; however, the apparatus was in the vertical position for both natural convection and combined forced and natural convection. Referring to Fig. 2, the diffuser section was replaced by a plug when natural convection experiments were performed to avoid the 'chimney effect'.

The inside diameter of the test section where the thermistor disks were mounted was 3.25 cm and the length of the velocity development section was 26.5 cm. Uniformity of velocity upstream of the heat transfer model was within 8% as measured by a pitot-tube traverse.† The air velocity was varied by controlling the inlet air flow rate. The free-stream air temperature was measured with a thermocouple probe and a variable d.c. power supply was used as the current source to self-heat the thermistor. Digital multimeters were used to measure simultaneously the voltage drop across the thermistor and standard resistor of known value, which, as shown in Fig. 1, was connected in series with the thermistor.

Experimental uncertainty in the Prandtl number is assumed to be negligible since it is primarily a function of air temperature, which was accurate to ± 0.6°C. Maximum experimental uncertainty in the Reynolds number was $\pm 7\%$, due primarily to uncertainty in the velocity measurements. Maximum experimental uncertainty in the Nusselt number was $\pm 12\%$, due primarily to uncertainty in the measured convective heat transfer coefficients. Uncertainties in geometry measurements were relatively small, and thus their only significant influence was in the measurement uncertainty of the convective heat transfer coefficient.

EXPERIMENTAL DATA AND RESULTS

As was mentioned earlier, six different disk heat transfer models were tested, the diameters ranging from 5.21 to 19.99 mm, and a thickness to diameter aspect ratio, t/d, from 0.062 to 0.2. The edges of the disk were relatively sharp (edge radius $\cong$ 0.04 mm). For the full range of measurements taken for forced, natural, and combined forced and natural convection, air velocities, v_f, ranged from 0.0015 to 35 m s^{-1} (0.005–114 ft s^{-1}), temperature differences, $T-T_f$, from 5 to 56°C (9–100°F) and the convective heat transfer coefficients, h, from 23 to 307 W m^{-2} °C^{-1} (4–54 Btu h^{-1} ft^{-2} °F^{-1}). Property values for air, which was at atmospheric pressure, were evaluated at the film temperature, T_{film}, where $T_{film} = (T_w + T_f)/2$. Reynolds numbers, Re_d, ranged from 2.0 to 3.0×10^4.

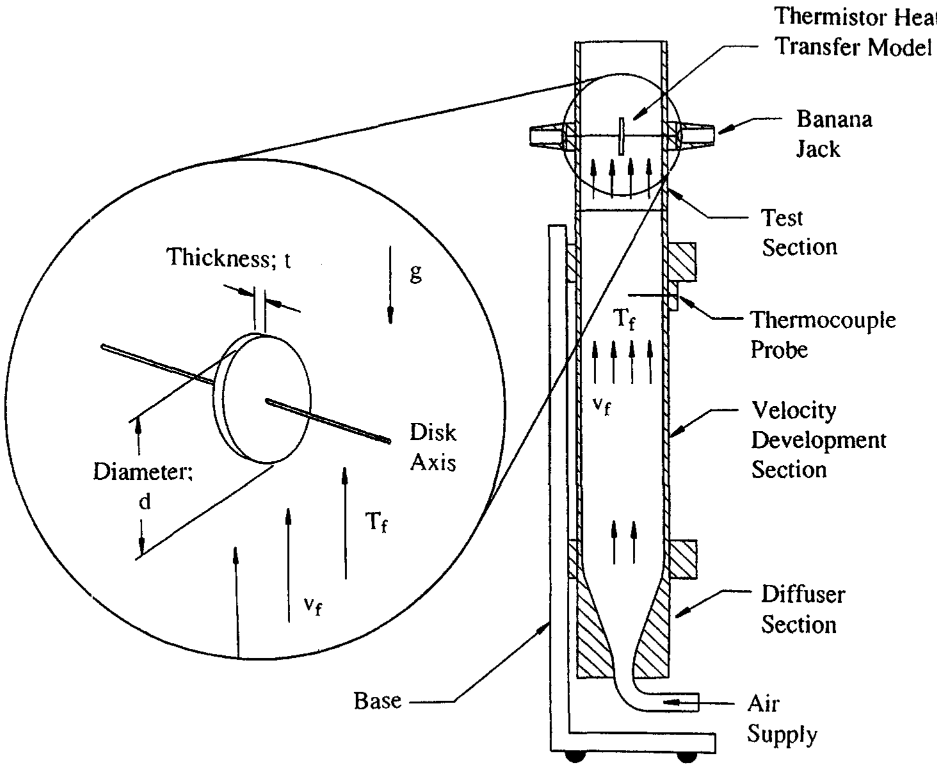

Fig. 2. Schematic of experimental apparatus and disk orientation.

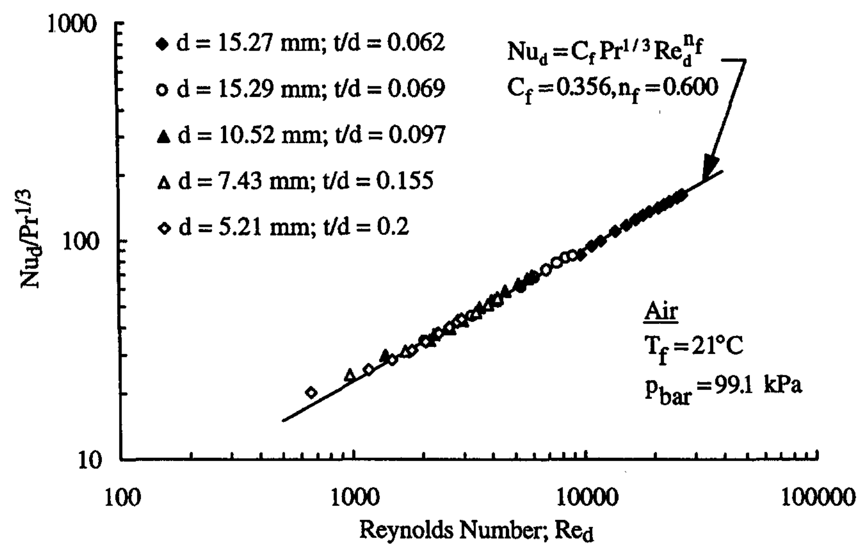

Fig. 3. Convective heat transfer for flow past a stationary circular disk whose axis is perpendicular to the flow.

Forced convection

The experimental results for forced convection are depicted in dimensionless form in Fig. 3, where $Nu_d/Pr^{1/3}$ is plotted as a function of Reynolds number, Re_d. Experiments yielding these data have been repeated many times, with excellent repeatability. The empirical correlation† which fits all of the data is given by

$$Nu_d = C_f Pr^{1/3} Re_d^{n_f} \quad C_f = 0.356 \; n_f = 0.600$$

$$9\times10^2 \leqslant Re_d \leqslant 3\times10^4. \quad (9)$$

The correlation coefficient for the curve fit is 0.998, with a maximum deviation of less than 6% and the form of equation (9) is the classic form for forced convection correlations for external flow over various geometries [1–5].

It should be pointed out at this time that only thin circular disks were tested (the thickness-to-diameter aspect ratio, t/d, varied from 0.058 to 0.20). For this range of aspect ratio, if any influence of the aspect ratio exists, it must be less than that of the experimental uncertainty in the measurements, since no discernible pattern of influence was observed for any data presented in this paper, regardless of the mode of heat transfer. Therefore, a range restriction on the aspect ratio cannot be made at this time.

As was discussed by Wedekind [7], an analysis of the experimental uncertainty in the indirect measurement of the convective heat transfer coefficient, due to uncertainties in the various direct measurements, indicates a maximum error of approximately 12%. From a statistical perspective, the measurement errors would normally be expected to be less than the maximum. This seems to be borne out in the relatively small level of scatter in the experimental data. One of the reasons for this relatively high level of accuracy is the technique of using thermistor disks as heat transfer models, and utilizing their associated resistance/temperature characteristics for measuring surface temperature and Joule heating for measuring the heat transfer rate. The range of Reynolds numbers given in equation (9) represented the limits of the existing experimental apparatus and heat transfer models, not necessarily the limit of the existing correlation.

Natural convection

The experimental results for natural convection are depicted in dimensionless form in Fig. 4, where Nu_d is plotted as a function of Rayleigh number, Ra_d ($Ra_d = Pr\ Gr_d$). An empirical correlation which fits all of the data is given by

$$Nu_d = C_n(Pr\,Gr_d)^{n_n} \quad C_n = 1.759 \; n_n = 0.150$$

$$10^2 \leqslant Pr\,Gr_d \leqslant 10^5. \quad (10)$$

The correlation coefficient for the curve fit is 0.953, with a maximum deviation of less than 15%. The results of an analysis of the experimental uncertainty is identical to that for the forced convection data, with a maximum uncertainty of less than 12%.

Hassani and Hollands [8] were, to the best knowledge of the authors, the only prior researchers to perform natural convection experimentation with a three-dimensional disk body. However, their investigation was limited to a single disk model of diameter, $d = 82$ mm, and a thickness-to-diameter aspect ratio, $t/d = 0.1$. Figure 5 illustrates the comparison between the data of Hassani and Hollands [8] and that of the current research. It should be noted that the experimental data of Hassani and Hollands,‡ which was modeled after the work of Chamberlain *et al.* [32], was obtained utilizing a *transient* measurement technique to measure indirectly the convective heat transfer rate. Referring to Fig. 5, it can be seen that the transiently-obtained data agreed very well with the steady-state data obtained in the current research. It should also be noted that the empirical correlation, as described by equation (10), seems to be valid over a wider domain than specified, again the domain representing the limit of the existing experimental apparatus. In fact, the correlation seems to be valid over virtually the entire range of the data presented by Hassani and Hollands, albeit under-predicting at the high end of the Rayleigh number.§ This under-prediction will be discussed in a later section when the modified Reynolds number, Re_d^*, is utilized. However, more experimentation would need to be done to determine the precise limits of the existing correlation.

Combined forced and natural convection

Combined forced and natural convection is complicated by the coupling of the inertial and buoyancy forces. The experimental data in Fig. 6 clearly demonstrate the relative influence of natural convection for two different diameter disks in assisting flow. As the Reynolds number decreases, natural convection becomes more dominant with $Nu_d/Pr^{1/3}$ asymptotically approaching that of pure natural convection as represented by the dashed lines which were obtained from the natural convection correlation, equation (10), for the particular disk at the same temperature. As the Reynolds number increases, forced

† It should be pointed out that there is a small difference between the current correlation and that reported by Wedekind [7]. Wedekind reported $C_f = 0.591$ and $n_f = 0.564$. Although the exponents are virtually identical, the constant in front is not. This difference was traced to a resistance/temperature measurement error in calibrating the thermistor heat transfer models.

‡ The experimental data of Hassani and Hollands [8] were represented by the Nusselt number based on the square root of the area and the Rayleigh number based on a characteristic length, H, which was a function of the height and the average periphery of the disk. In this way, the research of Hassani and Hollands attempted to collapse the disk data on a single curve with a variety of other geometries. From knowledge of the disk geometry, the experimental data were transformed to where the disk diameter, d, was the characteristic length.

§ The under-prediction of the correlation could be attributed to some turbulence in the developing boundary layer; however, the Grashof numbers obtained by Hassani and Hollands [8] are below that necessary for a transition to turbulence to occur, as pointed out by the investigation of Vitharana and Lykoudis [33] for vertical surfaces. It should be noted that even if there is no turbulence present, typical natural convection data for other geometries, such as flat plates and disks [1] are not linear on a logarithmic plot over the entire range of the Rayleigh number.

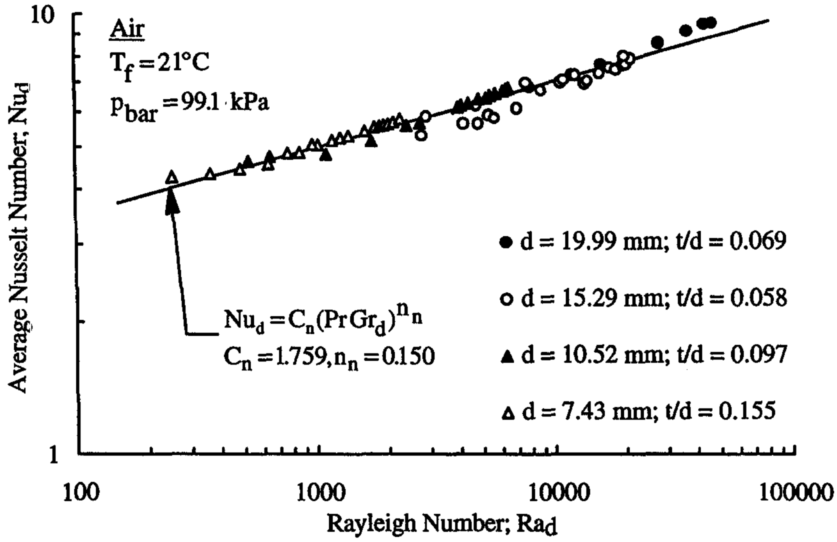

Fig. 4. Natural convective heat transfer for a stationary circular disk whose axis is perpendicular to the buoyancy force.

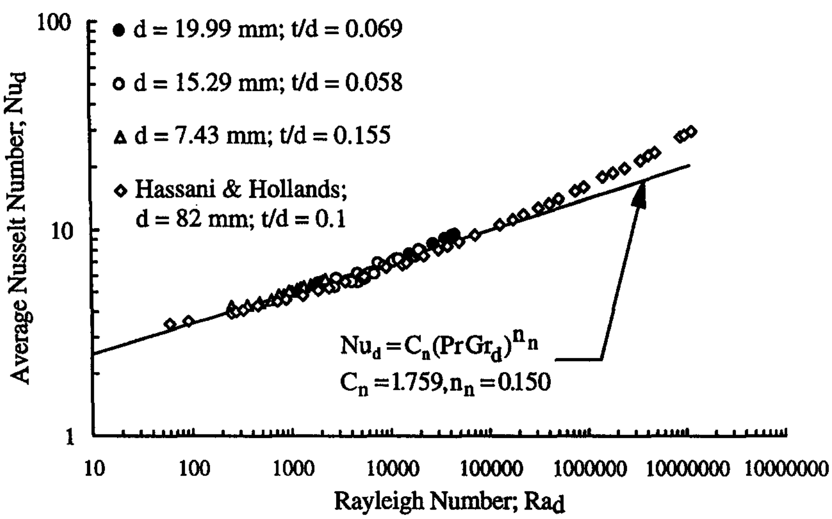

Fig. 5. Comparison between current natural convective heat transfer data for vertical disks and the experimental data of Hassani and Hollands [8].

convection becomes more dominant, asymptotically approaching pure forced convection.

Aside from the combining rule of equation (3), only a few other methods of dealing with combined forced and natural convection have received significant attention. One such method was the proposal of using a pseudo-velocity function to replace the buoyancy term in the governing differential equation [23, 24]. Although not specifically mentioned in the above referenced research, a pseudo-velocity gives rise to a modified Reynolds number which accounts for a buoyancy-induced velocity as well as the imposed free-stream velocity. Churchill [19] reviewed prior research, some of which postulated that an empirical correlation may be obtained using a modified Reynolds number, Re_d^*, which incorporates some characteristic buoyancy-induced velocity added vectorially to the imposed free-stream velocity. The concept of a modified Reynolds number also appears earlier in research presented by Lemlich and Hoke [34] and Hatton *et al.* [35], who investigated combined forced and natural convective heat transfer for horizontal cylinders.

For the present research, which is for assisting flow,

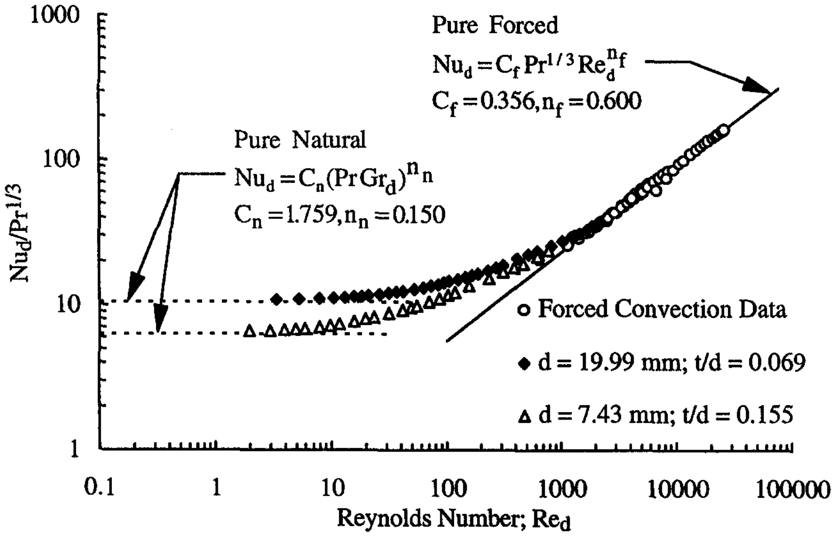

Fig. 6. Experimental data for pure forced and for combined forced and natural convective heat transfer for vertical disks.

the form of the modified Reynolds number, Re_d^*, utilizing a buoyancy-induced velocity averaged over the vertical centerline of the circular disk,† is

$$Re_d^* = Re_d + \lambda(Gr_d/2)^{1/2}. \quad (11)$$

This form of the modified Reynolds number‡ appears in several earlier investigations [19]. However, the particular choice of the characteristic buoyancy-induced velocity, which affects the weighting factor, λ, is somewhat unclear. It has been proposed that the local maximum velocity in the pure natural convection boundary layer be chosen as the characteristic velocity, as pointed out by a number of investigators§ [36]. Churchill [19] pointed to a number of researchers who took the weighting factor, λ, to be unity for geometries such as flat plates, cylinders and spheres. In an effort to obtain the appropriate weighting factor, the authors chose to use the maximum local velocity in the natural convection boundary layer averaged over the vertical centerline of the flat side of the disk.¶ Utilizing the integral solution to laminar natural convection boundary layer theory for a vertical flat plate [1, 4], the averaged maximum velocity in the natural convection boundary layer is

$$\bar{u}_{n,max} = \frac{1}{d}\int_{x=0}^{d} u_{n,max}(x)\,dx$$

$$= 0.511\left\{\left(\frac{20}{21}\right)+Pr\right\}^{-1/2}\left(\frac{\mu}{\rho}\right)\frac{Gr_d^{1/2}}{d} \quad (12)$$

where

$$u_{n,max}(x) = 5.17\left(\frac{\mu}{\rho}\right)\left\{\left(\frac{20}{21}\right)+Pr\right\}^{-1/2} \times\left\{\rho^2 g\beta'\frac{(T_w-T_f)x}{\mu^2}\right\}^{1/2}.$$

Thus, assuming uniform properties in the non-stratified air, the modified Reynolds number is expressed as

$$Re_d^* = \frac{\rho(u_f+\bar{u}_{n,max})d}{\mu}$$

$$= Re_d + \frac{0.723}{\left\{\left(\frac{20}{21}\right)+Pr\right\}^{1/2}}(Gr_d/2)^{1/2}$$

$$= Re_d + \lambda(Gr_d/2)^{1/2}. \quad (13)$$

The weighting factor, λ, is 0.558 for air at 21°C.

The experimental data for combined forced and natural convection were plotted in a form similar to that for forced convection data, except that the data are plotted against the modified Reynolds number

† In general, the average characteristic velocity would be obtained by utilizing the integral form of the mean-value theorem along the length of the vertical surface.

‡ It should be noted that the modified Reynolds number actually contains the Richardson number, which is generally accepted as the governing parameter for combined forced and natural convective heat transfer phenomena. Rearranging equation (11) yields $Re_d^* = Re_d\{1+\lambda(Ri_d/2)^{1/2}\}$.

§ Hall and Price [36] utilized the study by Cheesewright [37] and obtained $\lambda = 0.3$, but on a local basis only. It is noted that although the concepts of Hall and Price were useful to the current research, their study was for turbulent natural convection with a superimposed laminar free-stream velocity.

¶ This seemed the most plausible since the maximum velocity in the forced convection boundary layer, which is the imposed free-stream velocity, is used to define the Reynolds number. It is noted that disk geometry differs from a vertical flat plate and thus the characteristic velocity obtained from boundary layer theory is an approximation of the true velocity, although probably more accurate along the disk centerline.

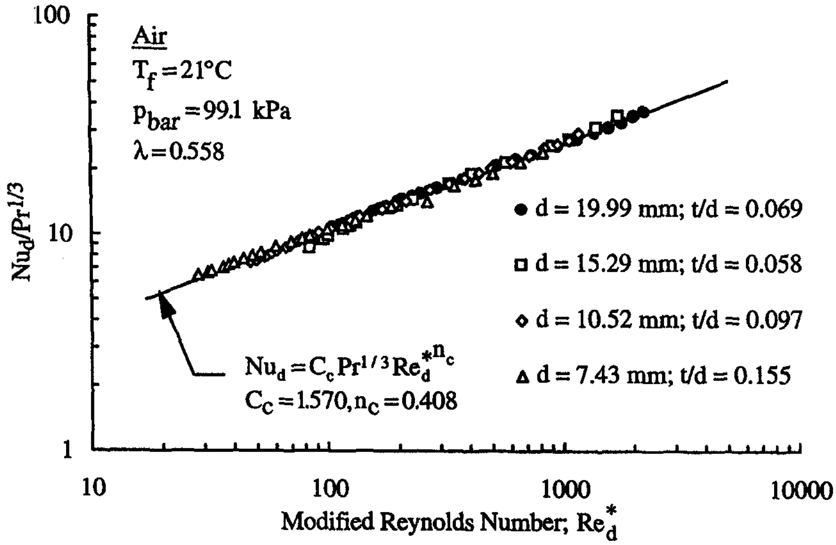

Fig. 7. Combined forced and natural convective heat transfer for a stationary circular disk whose axis is perpendicular to both free-stream velocity and the buoyancy force; assisting flow.

rather than the standard Reynolds number. Referring to Fig. 7, the result is seen to be very linear with excellent data collapse. The empirical correlation obtained for combined forced and natural convection† was

$$Nu_d = C_c Pr^{1/3} Re_d^{*n_c} \quad C_c = 1.570\, n_c = 0.408$$

$$70 \leqslant Re_d^* \leqslant 2100. \quad (14)$$

The correlation coefficient of the correlation is 0.983.

SUMMARY AND CONCLUSIONS

Experimental heat transfer data have been presented and dimensionless correlations proposed for forced, natural and (assisting) combined forced and natural convection from heated stationary isothermal circular disks over wide ranges of the Reynolds number, Rayleigh number and a modified Reynolds number, respectively. In the case of combined forced and natural convection, the modified Reynolds number, Re_d^*, was utilized which incorporates a buoyancy-induced characteristic velocity obtained from natural convection boundary layer theory which includes a weighting factor, λ, for the natural convection contribution. The modified Reynolds number is seen to reduce to the appropriate asymptotes of pure natural and pure forced convection.

An advantage to using the modified Reynolds number is that experimental data for forced, natural and combined forced and natural convection may be represented on the same graph. Referring to Fig. 8, the same dimensionless heat transfer data‡ are presented as in Figs. 3, 4 and 7 for forced, natural and combined forced and natural convective heat transfer, respectively, except that the data are plotted against the modified Reynolds number, Re_d^*, defined by equation (11), with $\lambda = 0.558$. Along with the experimental data, the previously developed empirical correlations for the three domains (forced, natural, and combined forced and natural convection) are superimposed. It should be noted that the forced convection data, and the corresponding empirical correlation, are virtually identical to those represented in Fig. 3 with the standard Reynolds number, because the natural convection contribution to the modified Reynolds number is negligible (less than 3% even at the low end of the forced convection data, where $Re_d^* = 2100$). Similarly, the standard Reynolds number in equation (11) is zero for pure natural convection and thus the modified Reynolds number is a function only of the Grashof number, as expressed by equation (10), where the characteristic velocity is buoyancy-induced. The pure natural convection correlation, equation (10), had to be slightly rearranged to be plotted against the modified Reynolds number. Thus:

$$\frac{Nu_d}{Pr^{1/3}} = C_n\, Pr^{-1/3}\, (Pr\, Gr_d)^{n_n}$$

$$= C_n \left(\frac{\sqrt{2}}{\lambda}\right)^{2n_n} Pr^{(n_n - 1/3)}\, Re_d^{*2n_n}. \quad (15)$$

The differences in the trends of the data for the three different domains (forced, natural, and combined forced and natural convection) seem clear, especially when the corresponding empirical correlations are superimposed. As can be seen from Fig. 8, a criterion may

† Before the discovery of the usefulness of the modified Reynolds number and the following empirical correlation, the authors attempted utilizing the combining rule of equation (2) with the experimental data resulting in a constant, n, of 3/2.

‡ Although the entire domain of all of the data is represented, some of the data are not displayed to eliminate over-crowding.

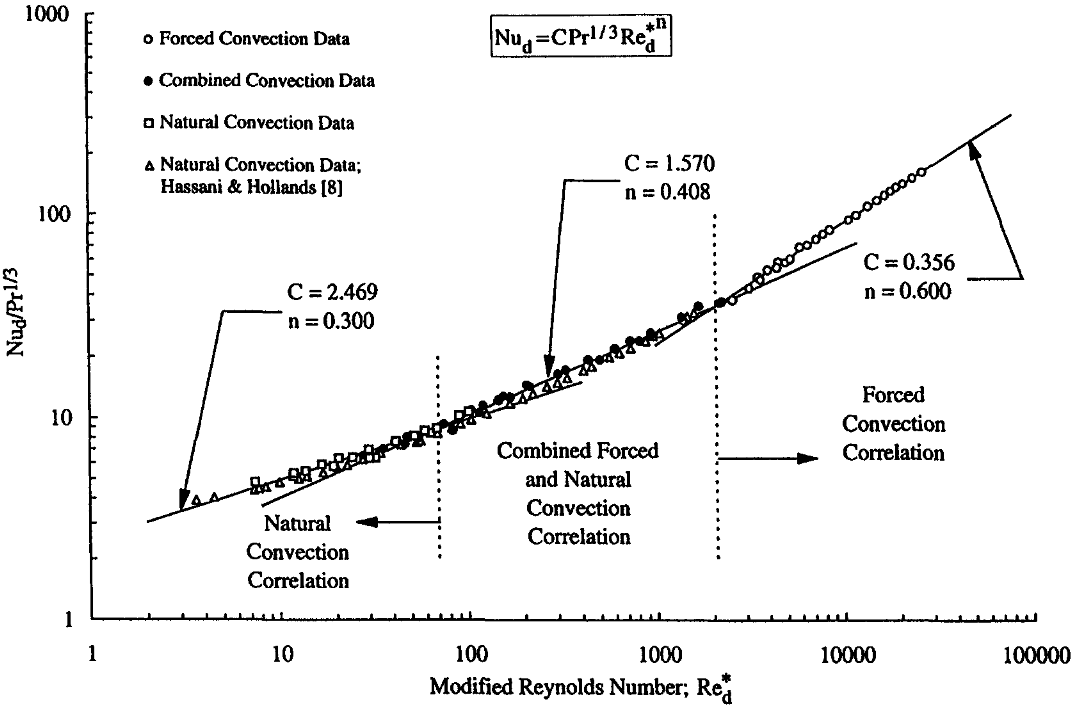

Fig. 8. Experimental results for forced, natural and combined forced and natural convective heat transfer from circular disks.

be established† between the different domains by equating the correlation for combined forced and natural convection with the correlation for pure forced and pure natural convection, respectively. Using this approach, it is seen that forced convection is dominant when $Re_d^* \geqslant 2100$, natural convection is dominant when $Re_d^* \leqslant 70$ and combined forced natural convection must be considered in between the two limits when $70 \leqslant Re_d^* \leqslant 2100$.

It should be pointed out that the established criterion was developed solely from equating the correlations as described above, and thus is not claimed to be a precise distinction of the three convection modes in any absolute sense. Referring again to Fig. 8, there is clearly some overlap in the current data between the natural and combined and between the combined and forced convection data. Also, when the pure natural convection data of Hassani and Hollands [8] were superimposed, the data were plotted right on top of the current data and overlapped most of what has been labeled as combined convection. The point being made here is that the boundaries between pure natural and combined convection, and between combined and pure forced convection, are somewhat 'blurred', the former apparently more than the latter.

† Sparrow and Gregg [38] developed criteria for pure forced convection in terms of the Richardson number for a vertical flat plate utilizing a numerical model. Such criteria may be more plausible in terms of the modified Reynolds number, Re_d^*. Similar criteria are mentioned in the research of Brdlik *et al.* [39].

Table 1. Summary of constants for use with general convective heat transfer correlation

Modified Reynolds number, Re_d^*	C	n
3–70	2.469	0.300
70–2100	1.570	0.408
2100–40 000	0.356	0.600

However, the value of representing the data in terms of the modified Reynolds number is that, as is apparent in Fig. 8, the correct convective heat transfer can be predicted regardless of the distinct convection mode by utilizing Table 1 with the general correlation

$$Nu_d = C\,Pr^{1/3}\,Re_d^{*n}. \qquad (16)$$

Since only air was tested, the correlations are recommended for Prandtl numbers near unity, which includes most common gases. The correlations may be valid for Prandtl numbers outside this range, however this is not known at this time since no experimental data are available. This is the subject of ongoing research.

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